

SSM2164 4-pole with pole-mixing

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This document explains the inner workings of the “Pole mixer” filter board, which uses a SSM2164 4-pole core along with a pole-mixer to realize 15 different transfer functions.

1 VCA-based 1-pole LPF

We remind here that a SSM2164 VCA cell works as a voltage-controlled current attenuator, according to the following equation:

$$i_{out} = i_{in} 10^{-\frac{3}{2}v_c}$$

Note that input pin of the SSM2164 is part of a differential pair whose other input is internally grounded (please refer to figure 23 in the SSM2164 datasheet), so the input pin is at ground potential and i_{in} is simply $\frac{v_{in}}{R}$.

Let us model the circuit in figure 1 from left to right. The current at the input of the VCA is the sum of the input current, and the current fed back from the output:

$$\begin{aligned} i_{in}(p) &= i_s(p) + i_o(p) \\ i_{in}(p) &= \frac{1}{R}(v_s(p) + v_o(p)) \end{aligned}$$

The op-amp is used as a current to voltage converter (or *transimpedance amplifier*), and is inverting:

$$\begin{aligned} v_o(p) &= -Z_{feedback}(p)i_-(p) \\ v_o(p) &= -\frac{1}{Cp}i_{out}(p) \\ v_o(p) &= -\frac{1}{Cp}10^{-\frac{3}{2}v_c}i_{in}(p) \end{aligned}$$

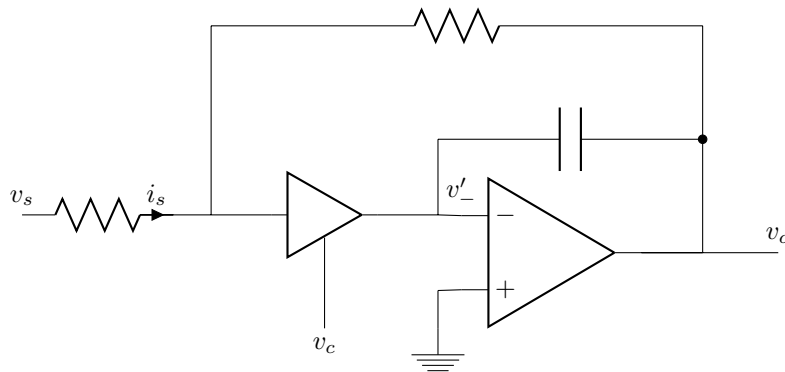


Figure 1: 1-pole voltage-controlled low-pass filter based on a VCA cell

$$v_o(p) = -\frac{1}{Cp} 10^{-\frac{3}{2}v_c} \frac{1}{R} (v_o(p) + v_s(p))$$

$$v_o(p) = \frac{-\frac{1}{RCp} 10^{-\frac{3}{2}v_c} v_s(p)}{\left(1 + \frac{1}{RCp} 10^{-\frac{3}{2}v_c}\right)}$$

We can now compute the transfer function:

$$H(p) = \frac{v_o(p)}{v_s(p)}$$

$$H(p) = \frac{-1}{1 + RC10^{\frac{3}{2}v_c}p}$$

This is the transfer function of an **inverting** 1-pole low-pass filter, with a pole at the frequency zeroing the denominator:

$$1 + RC10^{\frac{3}{2}v_c}2\pi f_c = 0$$

$$f_c = \frac{-1}{2\pi RC} 10^{-\frac{3}{2}v_c}$$

With $R = 33k\Omega$ (the datasheet suggests $R = 30k\Omega$ for the voltage to current conversion on the SSM2164 inputs), and $C = 220pF$, a frequency range of $22Hz$ to $22kHz$ can be obtained with v_c varying from $2.0V$ to $0.0V$ (the circuit doing this scaling is a simple op-amp in inverter configuration, not detailed here).

In the rest of this document we use the scaled variable $s = RC10^{\frac{3}{2}v_c}p$ to simplify the transfer function of this 1-pole cell into $\frac{-1}{1+s}$.

A simple 4-pole filter can be built by chaining 4 1-pole cells. The resulting transfer function is thus $\frac{1}{(1+s)^4}$.

2 Pole-mixing

Pole-mixing consists in mixing, with different gains, the output of each 1-pole cell of a 4-pole filter. Let us call a, b, c, d the gains applied to the output of the first, second, third and fourth cells ; and let us assume that the amplifier doing the summing and amplification is inverting. The resulting transfer function will thus be:

$$H_{a,b,c,d}(s) = \frac{a}{1+s} + \frac{-b}{(1+s)^2} + \frac{c}{(1+s)^3} + \frac{-d}{(1+s)^4}$$

Notice the alternating signs. This is because our SSM2164 based 1-pole cell is inverting, and this is very helpful indeed as we will see later! A little digression about this: In other pole-mixing circuits such as that of the Oberheim Xpander, additional op-amp stages are needed to buffer the output of each stage, and then to invert the output of each odd stage, requiring up to 6 op-amps to be added to the circuit. Here, the inversion and low-impedance output is built-in the low-pass cell itself, and the only thing to add for the mixing are resistors feeding into the virtual ground of a summing op-amp!

This "mixed" transfer function can be expanded into:

$$H_{a,b,c,d}(s) = \frac{as^3 + 3as^2 - 3as + a - bs^2 - 2bs - b + cs + c - d}{(1+s)^4}$$

Any 4th order transfer function with $(1+s)^4$ as its denominator:

$$H_{x,y,z,t}(s) = \frac{xs^3 + ys^2 + zs + t}{(1+s)^4}$$

can be implemented by pole mixing using the coefficients a, b, c, d which are solutions of these linear equations:

$$\begin{cases} a = x \\ 3a - b = y \\ 3a - 2b + c = z \\ a - b + c - d = t \end{cases}$$

A bank of resistors behind (a) multiplexer(s) can be used to switch sets of a, b, c, d coefficients. There are still a few problems with this solution though:

- It would require from 1 to 4 resistors per transfer function we want the filter to implement!
- Transfer functions like $\frac{s}{1+s}$ (1-pole HP) are not realizable with this system, since the order of the polynomial at the numerator can only be strictly lower than the order of the polynomial at the bottom.

A possible solution is to add a switch in the circuit to bypass the first low-pass cell (or more accurately use a lower integration cap to provide a very high cutoff-frequency). Using this approach, transfer functions of the form:

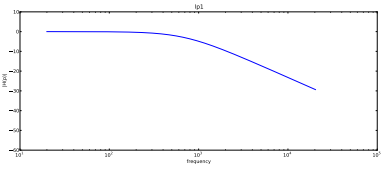
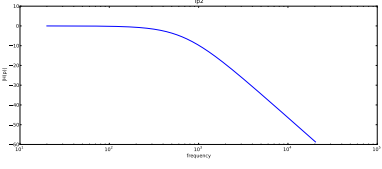
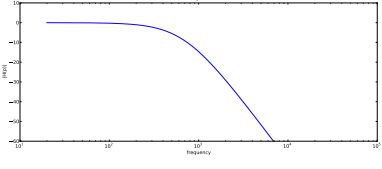
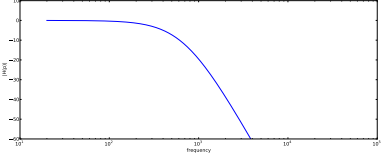
$$H_{x,y,z,t}(s) = \frac{xs^3 + ys^2 + zs + t}{(1+s)^3}$$

can also be implemented using the same circuit and the same design equations, with the only change that the first inverting 1-pole cell has been bypassed and replaced by a simple inverting cell.

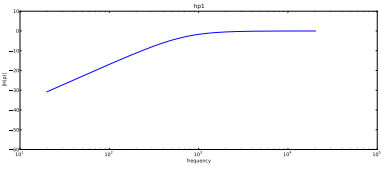
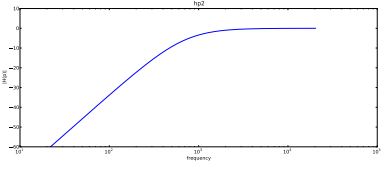
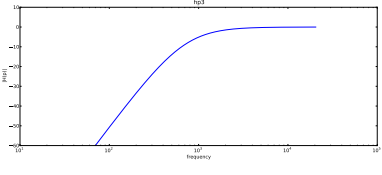
The second benefit of this “switched first 1-pole” approach is that it is often the case that if $H(s)$ is an interesting transfer function, $H(s)(1+s)$ will also be worth considering. Consequently, if a set of mixing resistors has been put in place for $H(s)$, it is trivial to just switch the first LP section off to realize $H(s)(1+s)$.

3 The 15 modes

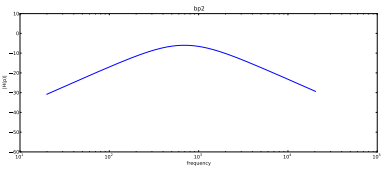
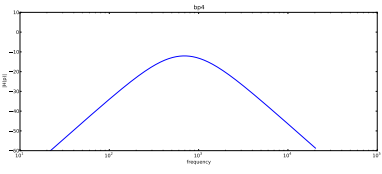
3.1 Low-pass modes

Name	Response	Transfer function	a	b	c	d	First LP stage
LP1		$\frac{1}{1+s}$	0	1	0	0	Disabled
LP2		$\frac{1}{(1+s)^2}$	0	1	0	0	Enabled
LP3		$\frac{1}{(1+s)^3}$	0	0	0	1	Disabled
LP4		$\frac{1}{(1+s)^4}$	0	0	0	1	Enabled

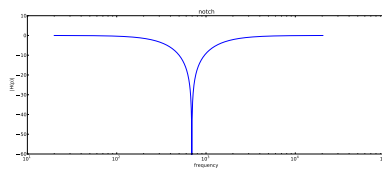
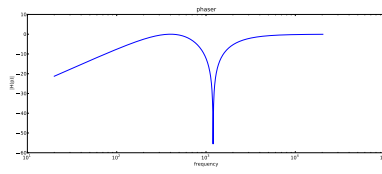
3.2 High-pass modes

Name	Response	Transfer function	a	b	c	d	First LP stage
HP1		$\frac{s}{1+s}$	1	1	0	0	Disabled
HP2		$\frac{s^2}{(1+s)^2}$	1	2	1	0	Disabled
HP3		$\frac{s^3}{(1+s)^3}$	1	3	3	1	Disabled

3.3 Band-pass modes

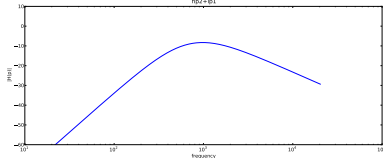
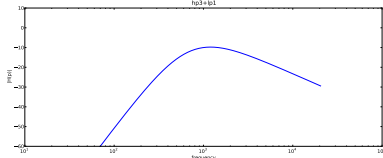
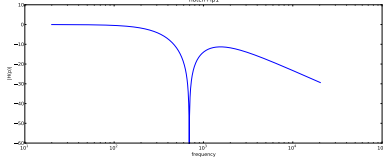
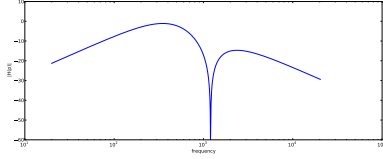
Name	Response	Transfer function	a	b	c	d	First LP stage
BP2		$\frac{s}{(1+s)^2}$	1	1	0	0	Enabled
BP4		$\frac{s^2}{(1+s)^4}$	0	1	2	1	Enabled

3.4 Notch and phaser modes

Name	Response	Transfer function	a	b	c	d	First LP stage
Notch		$\frac{s^3+s^2+s+1}{(1+s)^3}$	1	2	2	0	Disabled
Phaser		$1 - \frac{(1-s)^3}{(1+s)^3}$	1	3	6	4	Disabled

3.5 “Leftover” modes

These modes are of little interest, but are cheap to realize by toggling the LP stage on already existing modes.

Name	Response	Transfer function	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	First LP stage
HP2 + LP1		$\frac{s^3+s^2}{(1+s)^4}$	1	2	1	0	Enabled
			0	1	2	1	Disabled
HP3 + LP1		$\frac{s}{(1+s)^4}$	1	3	3	1	Enabled
Notch + LP1		$\frac{s^3+s^2+s+1}{(1+s)^4}$	1	2	2	0	Enabled
Phaser + LP1		$\frac{1}{1+s} - \frac{(1-s)^3}{(1+s)^4}$	1	3	6	4	Enabled